CORRECTION OF INSERTIONS/DELETIONS USING STANDARD CONVOLUTIONAL CODES AND THE VITERBI DECODING ALGORITHM

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Abstract: A new decoding scheme based on the Viterbi algorithm is presented, that can detect up to $n-1$ homogeneous bit insertions or deletions in an n-bit code symbol and re-establish synchronisation for a rate $R = k/n$ convolutional code sequence. The reversal errors occurring during the resynchronisation process can be corrected by an outer Reed-Solomon code, which is often concatenated in practice.

Key words: Convolutional Codes, Viterbi Decoding, Insertions and Deletions, Synchronisation.

1. INTRODUCTION

Error-correcting codes are generally designed to correct specifically additive errors, also known as reversal errors or substitution errors. On the other hand, bit-insertion/deletion errors occur when a bit is inserted into or deleted from a random position in the transmitted code sequence. Insertions/deletions may occur due to the loss of bit-synchronisation, which results in the loss of word-synchronization and long bursts of reversal errors in the decoded data sequence, possibly of infinite duration. Most research in the field of insertion/deletion correction has been performed for block codes, see e.g. [1-6].

As with block codes, insertions/deletions in convolutional codes cause the decoder to lose synchronisation, which results in long bursts of reversal errors in the decoded data until the decoder at the receiver regains synchronisation. The results pertaining to insertions/deletions in block codes cannot however be directly applied to convolutional codes, and alternative methods to detect and correct insertions/deletions for convolutional codes therefore involve either new encoding techniques as in [7], or new decoding techniques, for example employing the Levenshtein metric as in [8].

Although the topic of coding for insertion/deletion correction has been known since Levenshtein's landmark paper in the 1960's, surprisingly few papers in this difficult research field have appeared to date, especially when considering convolutional codes.

A decoding scheme for convolutional codes based on the well known Viterbi decoding algorithm was thus investigated that can detect closely spaced homogeneous bit-deletions or insertions occurring within any code symbol and re-establish synchronisation, and which can be easily incorporated into existing baseband communication systems such as digital magnetic or optical storage media [9, 10, 11], or metallic or optical cable systems [12], without having to design new coding schemes or convolutional codes.

We note with interest, as pointed out by an anonymous reviewer, that very specific error patterns which may perhaps also be modelled with insertions and deletions, arise when modems are used on bandpass radio channels, such as in wireless communications over fading channels. However, coding schemes for such channels are beyond the scope of this paper, and are perhaps an interesting topic for future research. This investigation was in fact motivated by one of the authors' previous exposure to, and current work on, baseband channels, see e.g. [9, 12, 13].

During the last decade, the problem of single or isolated insertion/deletion errors occurring on a magnetic or optical storage medium, as pointed out by practitioners in industry (see e.g. [14]), and leading to the corruption and loss of large data files, received some attention by researchers developing new coding schemes (see e.g. [15, 16]). This previous work had some limited success, usually due to the introduction of an undesirable amount of additional redundancy, which reduces the storage capacity of products in a highly competitive market.

In many baseband communication systems which may include cable systems as well as storage media, a modem is not used and the inner code on the channel is often a constrained code with a state system, which can be exploited with Viterbi decoding, see e.g. [9, 12]. In the past, it has also sometimes been proposed to use only one code, namely, a convolutional code, together with guided scrambling or partial response signaling on these severely bandlimited channels.

Furthermore, it is interesting to note that in the highly complex field of Information Theory, research on new or relatively unexplored topics, have often started with simple examples or models. For example, the single error correcting Hamming codes for additive errors represent a famous pioneering result leading to a large body of subsequent work. Other examples abound in the literature, see e.g. [17, 18], where results on single error correction for a simple channel model different than, but related to the work in this paper, was published.

We next present a brief overview of the rest of this paper: In Section 2, we discuss the new proposed decoding scheme. Section 3 follows with details on the simulations conducted. Some of the results obtained from the simulations are presented in Section 4, and Section 5 concludes with some of the limitations of this decoding scheme.

2. NEW PROPOSED DECODING SCHEME

The new decoding scheme for a rate $R = k/n$ convolutional code consists of n parallel decoders based on the Viterbi algorithm using the Hamming distance metric, as is standard practice [19, 20]. The bits received by each of the n decoders are framed in such a way, that each decoder processes bits one bit shifted with respect to the bits processed by the next decoder.

To determine which decoder is bit-synchronised, each of the decoders' rate of change for the lowest or best accumulated error metrics, namely:

$$\Delta\mu_i = \frac{\mu_i - \mu_{(i-\delta)}}{\delta}, \qquad (1)$$

is compared during each decoding iteration [21, 22]. The rate of change of accumulated error metric $\Delta\mu_i$ for the i'th decoder, is the difference between the current accumulated error metric, μ_i, and the accumulated error metric calculated a *decoding delay* number of iterations ago, $\mu_{(i-\delta)}$, divided by the decoder's decoding delay length, δ. Note that usually δ is set up to be five times the code's constraint length, or memory order m [19, 20, 23]. A decoder whose $\Delta\mu_i$ is lowest, is considered bit-synchronised, and the decoded data from this decoder's output is used.

For a channel in which, for example, only deletions occur, it is possible to detect up to $n-1$ consecutive or closely occurring deletions (such that the first and last deleted bit of a group are within $n-1$ bit positions of each other), in the received code sequence, by comparing the $\Delta\mu_i$ of n parallel decoders (for a $R = k/n$ convolutional code), and choosing the decoder with the lowest $\Delta\mu_i$. The detected deletions can then be transformed into short bursts of either erasures or reversal errors that can be subsequently corrected by an outer Reed-Solomon code, which is often concatenated in practical coding schemes with an inner convolutional code [20, 24]. After the detection of up to $n-1$ deletions, the framing of the n decoders are adjusted in order to detect possible future deletions.

This scheme can be set up in a similar way to function over an insertion channel.

3. SIMULATIONS

Simulations were conducted for 6 different convolutional codes, as listed in [19, 20]: three different rate $R = k/n$ convolutional codes with each rate having a small and large constraint length, or memory order, m, see Table 1.

Code Rate ($R = k/n$)	**Memory Order** (m)	**Free Distance** (d_{free})	**Generator Sequences** (octal)
1/2	2	5	$g_{11} = 7$ $g_{12} = 5$
1/2	6	10	$g_{11} = 634$ $g_{12} = 564$
1/3	2	8	$g_{11} = 5$ $g_{12} = 7$ $g_{13} = 7$
1/3	6	15	$g_{11} = 171$ $g_{12} = 133$ $g_{13} = 165$
1/4	2	10	$g_{11} = 5$ $g_{12} = 7$ $g_{13} = 7$ $g_{14} = 7$
1/4	6	20	$g_{11} = 564$ $g_{12} = 564$ $g_{13} = 634$ $g_{14} = 714$

Table 1: Properties of convolutional codes used in simulations

The binary symmetric deletion channel model shown in Fig. 1 was used for the simulations. The channel parameter probability values are such that the following equations hold:

$$p_0 + d_0 + r_0 = 1 \qquad (2)$$
$$p_1 + d_1 + r_1 = 1 \qquad (3)$$

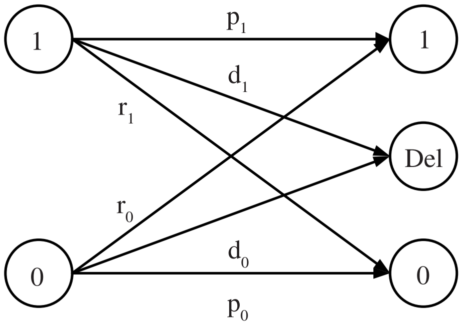

Figure 1: Simulation channel model

By assuming that the deletion probabilities are the same, and the reversal error probabilities are the same when a zero or a one is transmitted, the channel can be simplified as follows:

$$p_0 = p_1 = P_{cor}\,, \tag{4}$$

$$r_0 = r_1 = P_{err}\,, \tag{5}$$

$$d_0 = d_1 = P_{del}\,, \tag{6}$$

$$\therefore P_{cor} + P_{err} + P_{del} = 1\,. \tag{7}$$

where P_{cor} is the probability of correct transmission, P_{err} is the probability of a reversal/additive error, and P_{del} is the probability of a deletion error.

This model, which is very similar to the one described in [7, 25] was used, since no generally accepted channel model for insertions/deletions exists, as far as the authors are aware. Other channel models that have been used, were designed specifically for the research being conducted. For example, in [26], a generalised Gilbert-Elliot model was used. Another binary channel model in which substitution and insertion/deletion errors occur, was used in [27] and a similar model was used in [3], and a discrete probabilistic asynchronous channel was used in [28].

A block diagram of the complete simulation setup is shown in Fig. 2.

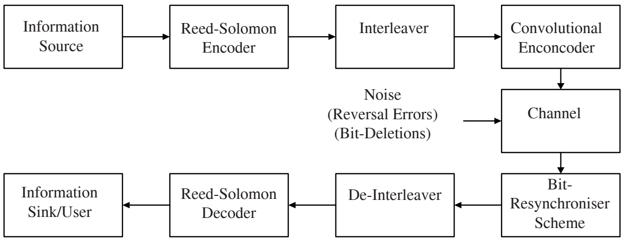

Figure 2: Simulation block diagram

4. RESULTS

Fig. 3 shows the ability of this scheme to detect statistically independent random deletions occurring with a probability P_{del}. No minimum guard space was enforced between the deletions, and no reversal errors were introduced into the code sequences. The graphs depict the scheme's deletion detection performance, P_{deldet}, for the 6 different convolutional codes used. From Fig. 3, it can be seen that for deletion probabilities less than approximately 10^{-4}, this scheme detects all the deletions occurring in all of the convolutional codes used, with a high probability, i.e. for $P_{del} < 10^{-4}$, $P_{deldet} \to 1$, for all of the codes used. It is evident that this scheme is better able to detect deletions when using convolutional codes with a larger n (as in $R = k/n$) and a smaller memory order, m. Fig. 4 shows the corresponding BER graphs for the $R = 1/4$ codes.

To illustrate how this scheme converts the detected deletions into correctable erasures or reversal errors, a simple (7,3) outer Reed-Solomon code was implemented, as shown in Fig. 2, that corrected the burst errors produced by the bit-resynchronisation. The block interleaving degree of the RS code was chosen to be commensurate with $5m$, i.e. the decoding delay of the Viterbi decoder.

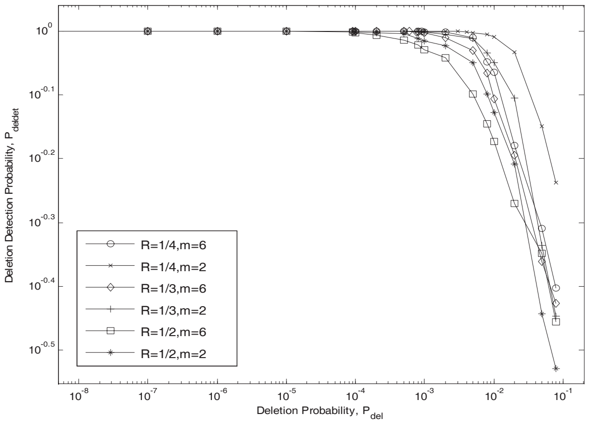

Figure 3: Deletion detection probabilities (Note: for $P_{del} < 10^{-4}$, $P_{deldet} \to 1$.)

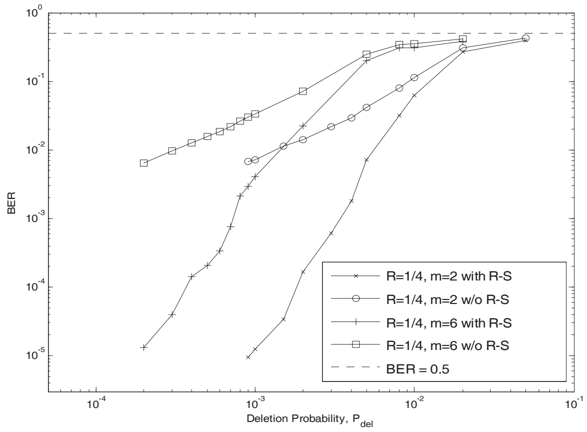

Figure 4:BER vs P_{del} for $R = 1/4$ convolutional code with and without concatenation with outer R-S codes

To ascertain the effects of background reversal errors on the ability of this scheme to detect deletions, random reversal errors were introduced into the transmitted code sequence with probabilities $P_{err} = 0.1$, 0.01 and 0.001.

From Figs. 5 and 6, it can be seen that high reversal error probabilities ($P_{err} > 0.01$), drastically affect the deletion detection performance of this scheme, while lower reversal error probabilities have very little effect on the ability of this scheme to reliably detect deletions and re-establish bit-synchronisation. The degree to which the reversal errors affect the performance of this scheme is largely dependent on the free distance d_{free} and memory order m, of the convolutional codes used.

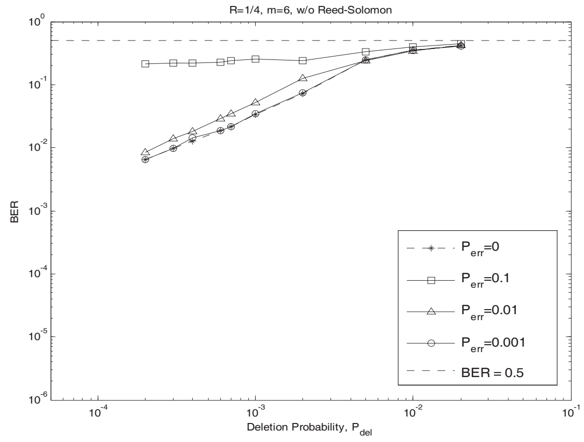

Figure 5: BER vs P_{del} for $R = 1/4$, $m = 6$, w/o R-S, for different reversal error probabilities

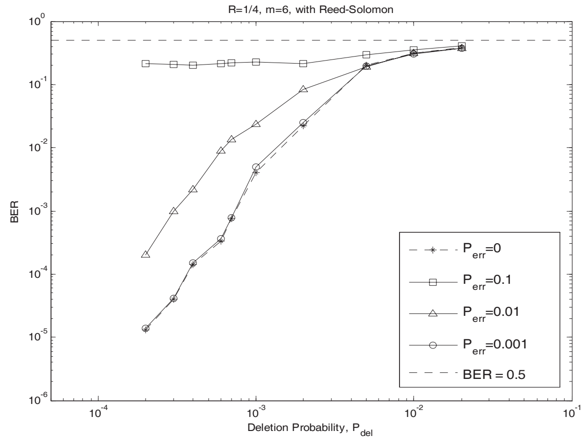

Figure 6: BER vs P_{del} for $R = 1/4$, $m = 6$, with R-S, for different reversal error probabilities

5. LIMITATIONS

If it is necessary to ensure that this scheme is able to always detect every pattern of up to $n-1$ consecutive deletions in a code symbol, a guard space of at least $5m$ n-bit symbols (corresponding to the decoding delay of the decoder) is required between the occurrences of groups of $n-1$ closely occurring deletions. Furthermore, this scheme is able to detect d consecutive deletions for all cases such that $d \bmod n \neq 0$, but can only re-synchronise for a deletions where $a = d \bmod n$.

It should again be noted that this coding scheme and the results in this paper only pertain to certain baseband systems and that further research and more complex schemes may be necessary to compensate for synchronisation failure in modems on bandpass channels.

6. CONCLUSION

A decoding scheme for convolutional codes that is able to detect insertions or deletions and re-establish bit-synchronisation is presented to augment and improve the performance of existing convolutional coding schemes. By implementing parallel Viterbi decoders using the standard Hamming metric, insertions or deletions are detected and then corrected by transforming them into burst erasures or reversal errors, which can be subsequently corrected by concatenating this scheme with outer Reed-Solomon codes.

With the exception of perhaps [14], very few results have been published on experimental studies covering the joint occurrence of reversal and other error types on channels. The recording channels considered in [14], indicate that reversals occur with probabilities much lower than $P_{err} = 0.001$ and deletions occur with probabilities several orders of magnitude lower. Under these conditions, the scheme proposed in this paper may perform exceedingly well.

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