This article will define the various elements of the South African Coordinate Reference System (SACRS) in detail and, in particular, distinguish between a coordinate system (projected) and a geodetic datum.

<table>
<thead>
<tr>
<th>Definitions*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate reference system</td>
</tr>
<tr>
<td>Datum</td>
</tr>
<tr>
<td>Easting (E)</td>
</tr>
<tr>
<td>Ellipsoid</td>
</tr>
<tr>
<td>Geodetic coordinate system</td>
</tr>
<tr>
<td>Geodetic datum</td>
</tr>
<tr>
<td>Map projection</td>
</tr>
<tr>
<td>Northing (N)</td>
</tr>
<tr>
<td>Projected coordinate system</td>
</tr>
<tr>
<td>Southing (x)</td>
</tr>
<tr>
<td>Vertical datum</td>
</tr>
<tr>
<td>Westing (y)</td>
</tr>
</tbody>
</table>

* All definitions from (ISO 19111:2007(E)), unless otherwise stated,
### Table 1: Coordinate difference of 30302S001 Hartebeesthoek VLBI 7232

<table>
<thead>
<tr>
<th>Point</th>
<th>Datum</th>
<th>Epoch</th>
<th>Time since</th>
<th>Position (Gauss Conform projection)</th>
<th>Central meridian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Reference Epoch</td>
<td>y (westing) m</td>
<td>x (southing) m</td>
</tr>
<tr>
<td>30302S001 Hartebeesthoek</td>
<td>ITRF91</td>
<td>1994.0</td>
<td>6.0</td>
<td>-86864.896</td>
<td>2864799.998</td>
</tr>
<tr>
<td>VLBI 7232</td>
<td>ITRF2005</td>
<td>2010.02</td>
<td>10.02</td>
<td>-86865.114</td>
<td>2864799.625</td>
</tr>
<tr>
<td>ITRF91 (epoch 1994.0) -</td>
<td></td>
<td></td>
<td>0.218</td>
<td>0.373</td>
<td>0.041</td>
</tr>
<tr>
<td>ITRF2005 (epoch 2010.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is important to be aware of the following:

- The Earth is constantly changing shape. To be understood in context, when the motion of the Earth’s crust is observed, it must be referenced. A Terrestrial Reference frame provides a set of coordinates of some points located on the Earth’s surface. It can be used to measure plate tectonics, regional subsidence or loading and/or used to represent the Earth when measuring its rotation in space. This rotation is measured with respect to a frame tied to stellar objects, called a celestial reference frame.

- The International Earth Rotation and Reference Systems Service (IERS) was created in 1988 to establish and maintain a Celestial Reference Frame, the ICRF, and a Terrestrial Reference Frame, the ITRF. The Earth Orientation Parameters (EOPs) connect these two frames together. These frames provide a common reference to compare observations and results from different locations.

- The ITRF is constantly being updated. Eleven realisations of the ITRS were set up from 1988. The latest is the ITRF2008 (June 2010). All these realisations include station positions and velocities.

- Hartebeesthoek94 coordinates do not take cognisance of the velocities associated with stations contributing to ITRF91 and other realisations.

- The coordinates of the Hartebeesthoek94 datum are, therefore, locked in time at the given epoch, being 1994.0.

- Hartebeesthoek94 coordinates may be transformed to other realisations of ITRF by accessing the coordinates of fiducial ITRF stations and their associated velocity vectors, published by the IERS, at the epoch of interest.

The extent of the difference between these two reference frames (in South Africa) can be measured...
be gauged by comparing the coordinates of the only fiducial station that has existed since the introduction of Hartebeesthoek94 (VLBI 7232), which yields the difference seen in Table 1.

- This transformation can also be achieved by applying a seven parameter Helmert transformation. See http://lareg.ensg.ign.fr/ITRF/.

Hartebeesthoek94 and the WGS84 Reference Frame

Key facts relating to Hatebeesthoek94 and the WGS84 Reference frame are outlined below:

- The World Geodetic System 1984 (WGS84) is a Conventional Terrestrial Reference System that includes in its definition a reference frame, a reference ellipsoid, a consistent set of fundamental constants, and an Earth Gravitational Model (EGM) with a related global geoid [4].

- The global geocentric reference frame and collection of models known as the World Geodetic System 1984 Reference Frame (WGS84RF) has evolved significantly since its creation in the mid-1980s. The WGS84RF has been redefined periodically.

- GPS satellite orbits and control segment positions operate in the WGS84RF.

- The WGS84RF should not be confused with the WGS84 ellipsoid.

- Since 1997, the WGS84RF has been maintained within 10 cm, and more recently within 5 cm of the current ITRF. The latest realisation of the WGS84RF is G1150 [5].

- Hence, the differences between Hartebeesthoek94 and the WGS84RF would be of the same magnitude as Hartebeesthoek94 and the current ITRF realisation (see “Hartebeesthoek94 and other ITRF realisations” above).

Connecting/referencing to Hartebeesthoek94

There are two methods for a point/data to be referenced to Hartebeesthoek94 datum:

- Direct connection: the position/s must be determined relative to any point in the national control survey network (horizontal), such as the 29 000 trigonometrical beacons and 20 000 town survey marks. This would constitute direct connection.

- Indirect connection: can be achieved by determining positions relative to points that have already been directly connected.

Note: When data is collected using autonomous GPS (which operates in the WGS84RF, and has a typical accuracy of 5 m), it can be deemed to be referenced to Hartebeesthoek94. This is because the uncertainty in position is an order of magnitude larger than the difference in position of a point in the respective datums.

When using real-time TrigNet services (which is referenced to ITRF2005), users will have to occupy points referenced to Hartebeesthoek94 to establish a localised relationship (calibration) between the respective datums. This is a requirement especially in cadastral surveys.

<table>
<thead>
<tr>
<th>Name</th>
<th>Areas used</th>
<th>Central meridian(s)</th>
<th>Latitude of origin</th>
<th>CM Scale factor</th>
<th>Zone width</th>
<th>False easting at origin</th>
<th>False northing at origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse Mercator</td>
<td>Various, worldwide</td>
<td>Various</td>
<td>Various</td>
<td>Various</td>
<td>Usually less than 6°</td>
<td>Various</td>
<td>Various</td>
</tr>
<tr>
<td>Gauss-Conform (Transverse Mercator south oriented)</td>
<td>South Africa</td>
<td>2° intervals E of 11°E</td>
<td>0°</td>
<td>1</td>
<td>2°</td>
<td>0 m</td>
<td>0 m</td>
</tr>
<tr>
<td>UTM North hemisphere</td>
<td>Worldwide</td>
<td>6° intervals E &amp; W of 3° E &amp; W</td>
<td>Always 0°</td>
<td>Always 0.9996</td>
<td>Always 6°</td>
<td>500 000 m</td>
<td>0 m</td>
</tr>
<tr>
<td>UTM South hemisphere</td>
<td>Worldwide</td>
<td>6° intervals E &amp; W of 3° E &amp; W</td>
<td>Always 0°</td>
<td>Always 0.9996</td>
<td>Always 6°</td>
<td>500 000 m</td>
<td>10 000 000 m</td>
</tr>
<tr>
<td>Gauss-Kruger</td>
<td>Former USSR, Germany, S. America</td>
<td>Various, according to area of cover</td>
<td>Usually 0°</td>
<td>Usually 1.000000</td>
<td>Usually less than 6°, often less than 4°</td>
<td>Various but often 500 000 prefixed by zone number</td>
<td>Various</td>
</tr>
</tbody>
</table>

Table 2: Different forms of the Transverse Mercator Projection.
The Transverse Mercator projection

Johann Heinrich Lambert was a German/French mathematician and scientist. His mathematics was considered revolutionary for its time and is still considered important today. In 1772 he released both his Conformal Conic projection and the Transverse Mercator projection. The Transverse Mercator projection is the transverse aspect of the Mercator projection, which is a cylindrical projection, turned about 90° so that the projection is based on meridians and not the parallels (see Figs. 2 and 3).

The Transverse Mercator projection, in its various forms, is the most widely used projected coordinate system for world topographical and offshore mapping. All versions (e.g. Gauss Conform, Gauss Kruger, and Universal Transverse Mercator) have the same basic characteristics and formulas.

The differences which distinguish the different forms of the projection, and which are applied in different countries arise from variations in the choice of the coordinate transformation parameters, namely the latitude of the origin, the longitude of the origin (central meridian), the scale factor at the origin (on the central meridian), and the values of false easting and false northing, which embody the units of measurement, given to the origin. Additionally there are variations in the width of the longitudinal zones for the projections used in different territories.

Table 2 indicates the variations in the projection parameters which distinguish the different forms of the Transverse Mercator projection:

Part 2 of this article will focus on the Gauss Conform Coordinate System, projection formulae and the South Africa Coordinate Reference System. It will be published in the Jan/Feb 2012 edition of PositionIT.

References


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The South African Coordinate Reference System (Part 2)

by Aslam Parker, Chief Directorate: National Geo-spatial Information

Part 1 of this article (see PositionIT Nov/Dec 2011) focused on the Hartebeesthoek94 Datum and the Transverse Mercator Projection. Now the focus turns to the Gauss Conform Coordinate System and the definition of the South African Coordinate Reference System (SACRS).

The Gauss Conform Coordinate System (as used in South Africa) uses the Transverse Mercator map projection formulae modified to produce westings ($y$) and southings ($x$) instead of northings ($N$) and eastings ($E$). Note that the Gauss Conform projection is used in the southern hemisphere only.

This projection is used for the computation of the plane westings ($y_{Lo}$) and southings ($x_{Lo}$) coordinates, commonly (but incorrectly) known as the "Lo coordinate system".

Coordinate system conventions

Outlined below are details on coordinate system conventions:

Reference longitude / central meridian (zone/belt)
- These 2° longitude wide zones (belts) are centred on every odd meridian, i.e. (15°E, 17°E, ... 35°E as well as 37°E for (Marion and Prince Edward Islands) as central meridian. Example; Longitude 19°E is the central meridian (CM) of the belt between 18°E and 20°E.
- The origin of each zone is the intersection of each uneven degree of longitude (longitude of origin = $Lo$) and the equator.
- Each zone is named after the longitude of origin i.e. $Lo$ 17°, $Lo$ 19°, $Lo$ 21° etc. and is independent of geodetic datum.

Latitude at natural origin / reference latitude
The equator 0°E, is the latitude of reference or origin of the Gauss Conform Coordinate System.

$x$ (Southings)
- Coordinates are measured southwards from the equator
- Increases from the equator (where $x = 0$ m) towards the south pole (with a maximum of ±3 840 000 m for continental South Africa).
- Similar to the "northing" coordinates but sign in opposite.

$y$ (Westings)
- Coordinates are measured from the central meridian ($Lo$) of the respective zone.
- Increases from the CM (where $y = 0$) in a westerly direction.
- "$y$" is +ve west of the CM and −ve east of the central meridian.
- Since the zone width is 2° (1°) either side of the central meridian, the "$y$" value should range between +105 000 m and ±105 000 m in South Africa.
- Unless specifically intended, a feature with a "$y$" ordinate exceeding the abovementioned values should be referenced to the adjacent central meridian.

False origin
There is no false origin ($y = 0$ m at central meridian and $x = 0$ m at equator).
Distortion

- Infinitesimally small circles of equal size on the globe appear as circles on the map (indicating conformality) but increase in size away from the central meridian (indicating area distortion).
- The central meridian is the only line of longitude that is a straight line on the map.
- The equator is the only line of latitude that is a straight line on the map.
- There is a scale distortion that increases from zero as you go away from the central meridian. i.e. If points A and B are far from the central meridian, and you walk from A to B and find the distance to be x metres. Then you will find that the distance as shown by the map will not be exactly x metres.
- This significant distortion of scale as you move away from the central meridian was the key reason for limiting zone width to 2°.

Projection formulae

A detailed explanation of projection formulae [7] can be seen in Figs. 7 and 8. Fig. 7 outlines the conversion of Geographical coordinates to Gauss Conform coordinates and Fig. 8 outlines the conversion of Gauss Conform Coordinates to Geographic coordinates. Sample coordinates reflecting these conversions can be seen in Table 3.

The South African Coordinate Reference System

It must be stressed that any position reported in the SACRS must be referenced to both the Hartebeesthoek94 datum and the Gauss Conform Coordinate System as defined earlier. Any position reported in other projections, (e.g. the standard Transverse Mercator or UTM projection), or another datum (e.g. Cape Datum) would, by definition, not be referenced to the SACRS.

To summarise: SACRS = Gauss Conform Coordinate System (south oriented version of standard Transverse Mercator Projection) referenced to the Hartebeesthoek94 Datum. See Figs. 9 (a) and (b).

Defining the South African Coordinate Reference System in software

Defining Hartebeesthoek94 Datum within your GIS/GNSS software:
- Choose ellipsoid as WGS84
- Assign datum name as Hartebeesthoek94
Conversion of degrees to radians and vice-versa:

\[ \text{Rad} = \text{Deg} \times \frac{\pi}{180} \quad \text{Deg} = \text{Rad} \times \frac{180}{\pi} \]

**Conversion of Geographical coordinates \((\varphi, \lambda)\) to Gauss Conform coordinates \((y, x)\):**

- \(a\) = semi-major axis of the reference ellipsoid in metres.
- \(b\) = semi-minor axis of the reference ellipsoid in metres.

**Given:**
- \(\varphi\) (Latitude in degrees decimal, positive south)
- \(\lambda\) (Longitude in degrees decimal, positive east)
- \(Lo\) \((\lambda_0)\) (Reference longitude/ central meridian in integer degrees)

**Find:**
- \(y\) (Gauss Conform ordinate in metres, westing)
- \(x\) (Gauss Conform ordinate in metres, southing)

Convert \(\varphi, \lambda_0, \lambda\) to radians

\[ \varphi = \text{abs}(\varphi) \quad (\text{i.e. use the absolute value of the latitude}) \]

\[ e^2 = \frac{a^2 - b^2}{a^2} \]

\[ e'^2 = \frac{a^2 - b^2}{b^2} \]

\[ \eta^2 = e^2 \cos^2 \varphi ; \quad \tau = \tan \varphi \]

\[ A = 1 + \frac{3}{4} e^2 + \frac{45}{64} e^4 + \frac{175}{256} e^6 + \frac{11025}{16384} e^8 + \frac{43659}{65536} e^{10} \]

\[ B = \frac{3}{4} e^2 + \frac{15}{16} e^4 + \frac{525}{512} e^6 + \frac{2205}{2048} e^8 + \frac{72765}{65536} e^{10} \]

\[ C = \frac{15}{64} e^4 + \frac{105}{256} e^6 + \frac{2205}{4096} e^8 + \frac{10395}{16384} e^{10} \]

\[ D = \frac{35}{512} e^6 + \frac{315}{2048} e^8 + \frac{31185}{131072} e^{10} \]

\[ E = \frac{315}{16384} e^8 + \frac{3465}{65536} e^{10} \]

\[ F = \frac{693}{131072} e^{10} \]

\[ B_s = a \left[ 1 - e^2 \right] \left( \frac{A \phi - \frac{B}{2} \sin 2\phi + \frac{C}{4} \sin 4\phi - \frac{D}{6} \sin 6\phi + \frac{E}{8} \sin 8\phi - \frac{F}{10} \sin 10\phi} {\sqrt{1 - e^2 \sin^2 \phi}} \right) \]

\[ \ell = \lambda_0 - \lambda \quad (\text{where } \lambda_0 \text{ is the longitude of the central meridian}) \]

\[ N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \]

\[ x = B_s + \frac{\ell^2}{2} N \sin \phi \cos \phi + \frac{\ell^4}{24} N \sin \phi \cos^2 \phi \left( 5 + 9 \eta^2 + 4 \eta^4 - \tau^2 \right) \]

\[ + \frac{\ell^6}{720} N \sin \phi \cos^2 \phi \left( 61 - 58 \tau^2 + 10 \eta^2 + 270 \eta^4 + 330 \eta^6 + 330 \eta^8 - 330 \eta^{10} \right) + \ldots \]

\[ y = \ell N \cos \phi + \frac{\ell^3}{6} N \cos^3 \phi \left( 1 + \eta^2 - \tau^2 \right) \]

\[ + \frac{\ell^5}{120} N \cos^3 \phi \left( 5 - 18 \tau^2 + \tau^4 + 14 \eta^2 - 58 \eta^4 - 330 \eta^6 - 330 \eta^8 + 330 \eta^{10} \right) + \ldots \]

**Note:** The Gauss Conform system is used in the southern hemisphere only.

Fig. 7: Conversion of Geographical coordinates to Gauss Conform coordinates [7].
Conversion of Gauss Conform coordinates \((y, x)\) to Geographical coordinates \((\phi, \lambda)\)

- \(a\) = semi-major axis of the reference ellipsoid in metres.
- \(b\) = semi-minor axis of the reference ellipsoid in metres.

**Given:**
- \(y\) (Gauss Conform ordinate in metres, westing)
- \(x\) (Gauss Conform ordinate in metres, southing)
- \(\text{Lo} (\lambda_0)\) (Reference longitude/ central meridian in integer degrees)

**Find:**
- \(\phi\) (Latitude in degrees decimal, positive south)
- \(\lambda\) (Longitude in degrees decimal, positive east)

**Formulae:**

- Convert \(\lambda_0\) to radians
  \(e^2 = \frac{a^2 - b^2}{a^2}\)
  \(c^2 = \frac{a^2 - b^2}{b^2}\)

- \(n = \frac{a - b}{a + b}\)

- \(\sigma = \frac{x_0 (1 + n)}{\frac{1}{4} n^2 + \frac{1}{4} n^4}\)

- \(\phi_f = \sigma + \frac{3}{2} \left( n - \frac{9}{16} n^3 \right) \sin 2\sigma + \frac{21}{16} n^2 \sin 4\sigma + \frac{151}{96} n^3 \sin 6\sigma\)

- \(\tau_f = \tan \phi_f\)

- \(\eta_f^2 = e^2 \cos^2 \phi_f\)

- \(M_f = \frac{a \left( 1 - e^2 \right)}{\left( 1 - e^2 \sin^2 \phi_f \right)^{\frac{1}{2}}}\)

- \(N_f = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi_f}}\)

- \(\phi = \phi_f - \frac{\eta^2}{2 M_f N_f} + \frac{\tau^2}{24 M_f N_f} \left( 5 + \eta^2 + 3 \eta^4 \right) + \frac{\tau^2}{24 M_f N_f} \left( 5 + 6 \eta^2 + 6 \eta^4 \right)\)

- \(\ell = \frac{y}{N_f \cos \phi_f} - \frac{y^3}{6 N_f^2 \cos \phi_f} \left( 1 + \eta^2 + 2 \eta^4 \right) - \frac{y^5}{120 N_f^3 \cos \phi_f} \left( 5 + 6 \eta^2 + 28 \eta^4 + 24 \eta^6 \right)\)

- \(\phi = -\phi\) (i.e. make the sign of the latitude negative, for the southern hemisphere)
- \(\lambda = \lambda_0 - \ell\) (where \(\lambda_0\) is the longitude of the central meridian)

**Software catering for "south oriented systems"** (see Fig. 11).

- If the software caters for the "South Azimuth system" (0° = south) and has the option of coordinates increasing in south and westerly direction, enable these options.
- Central latitude is equator (0° N/S)

**Defining the Gauss Conform Coordinate System within your GIS/GNSS software**

- Central longitude is the central meridian of the zone e.g 27°00’00”E
- False northing = 0
- False easting = 0
- Scale factor = 1
- The name that you assign the coordinate system is arbitrary, but common practice is to use the “WG/Lo” denotation.

Fig 8: Conversion of Gauss Conform coordinates to Geographical coordinates [7].
Fig. 9 (a): Current SACRS definition.

South African Coordinate Reference System (current)

<table>
<thead>
<tr>
<th>Geodetic Datum (Harbebeethekoek84)</th>
<th>Coordinate System (Gauss Conformal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WGS84 ellipsoid</td>
<td>Transverse Mercator Projection</td>
</tr>
<tr>
<td>Coincident with TRF91 epoch 1984.0</td>
<td>Coordinate System Conventions</td>
</tr>
</tbody>
</table>

Fig. 9 (b): SACRS definition prior to January 1999.


<table>
<thead>
<tr>
<th>Geodetic Datum (Cape Datum)</th>
<th>Coordinate System (Gauss Conformal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modified Clarke 1880 ellipsoid</td>
<td>Transverse Mercator Projection</td>
</tr>
<tr>
<td>Origin at Buffelsfontein</td>
<td>Coordinate System Conventions</td>
</tr>
</tbody>
</table>

Fig. 10: Defining Hartebeesteelhoek94 Datum.

Fig. 11: Defining the Gauss Coordinate System with software catering for south oriented systems.

Software that does not cater for the “South Azimuth system” (0° = South) (see Fig. 12).

Strictly speaking, in this case, one cannot reference projects to the SACRS, since the Gauss Conform Coordinate System cannot be defined. The closest option would be to use the standard Transverse Mercator Projection. This will result in northings and eastings instead of southings and westings. The coordinates will be identical in magnitude, but opposite in sign (see Fig. 12).

- Projection = Transverse Mercator
- Central latitude is equator (0° N/S)
- Central longitude is the central meridian of the zone e.g 19°00'00"E
- Scale factor = 1
- False northing = 0
- False easting = 0

Until users apply pressure to major vendors to implement "south oriented" coordinate systems, the SACRS cannot be correctly defined.

References


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